Topics in Topology and Complex Manifolds – MATH 995
Representation theory of compact Lie groups
Professor Gerald Hoehn – Spring 2008

We will explain the representation theory of compact Lie groups.

A *Lie group* is a differentiable manifold together with a compatible group structure on it. It is remarkable, that all such objects can be described explicitely if one assumes that the manifold is *compact and simply connected:* They are finite products of groups derived from the "classical groups" $\mathbf{SO}(n)$, $\mathbf{SU}(n)$, and $\mathbf{Sp}(n)$ or five "exceptionell groups" of dimensions 14, 52, 78, 133 and 248.

An important concept in mathematics is the notion of a *linear representation* of a group. For *compact* Lie groups it is enough to study finite dimensional linear representations. Again remarkably, it is enough to know only the representations of the n-dimensional torus $\mathbf{T}^n$ and of the group $\mathbf{SU}(2)$ explicitly.

The interest in compact Lie groups does not only arise from the attractiveness of the theory, but also from their central rôle in modern mathematics, like for topology, differential geometry, number theory or the theory of finite groups. They are also fundamental for applications in physics in particular theoretical mechanics, quantum mechanics, or the standard model of elementary particles as well as for modern developments like conformal field theory or string theory.

Prerequisites for this course are algebra and topology classes. The 700 level should be enough. Please see Prof. Hoehn in case there are questions.

Text book:

T. Bröcker and T. tom Dieck: *Representations of Compact Lie groups.*