Topics in Geometry – MATH 995
Mathieu Moonshine and K3 surfaces
Professor Gerald Hoehn – Fall 2014

Mathieu Moonshine refers to a new and unexpected connection between the Mathieu group M_{24} — one of the 26 sporadic groups — and the Weierstrass $\wp$-function.

There is a similar connections between the Monster — the largest of the 26 sporadic groups — and the j-invariant of elliptic curves which is called Monstrous Moonshine and was finally explained by the invention of vertex operator algebras and generalized Kac-Moody algebras. For Mathieu Moonshine, there is no such explanation yet, but there are indications that it is related to finite group actions on K3-surfaces. K3-surfaces are complex surfaces which are diffeomorphic to the Fermat quartic $\{(x_0 : x_1 : x_2 : x_3) \in \mathbf{C}P^3 \mid x_0^4 + x_1^4 + x_2^4 + x_3^4 = 0\}$.

In the lecture, I will give a quick introduction to modular forms and the Mathieu group M_{24} to formulate Mathieu Moonshine. Then I will discuss finite group action on manifolds and the equivariant Atiyah-Singer index theorem. Finally, I will concentrate on symplectic automorphism groups of K3 surfaces and prove Mukai's theorem relating them to the Mathieu group M_{23} and the connection to Mathieu Moonshine.

Prerequisites for this course are 800 level algebra and topology classes.

References:

Overview articles on Monstrous Moonshine:

R. Borcherds: *What is moonshine?*, arXiv:math/9809110.

T. Gannon: *Monstrous Moonshine: The first twenty-five years*, arXiv:math/0402345.

Some geometric aspects of Mathieu Moonshine:

T. Creutzig and G. Höhn: *Mathieu Moonshine and the Geometry of K3 Surfaces*, arXiv:1309.2671.

For the complex elliptic genus:

G. Höhn: *Komplexe elliptische Geschlechter und S^1-q̈uivariante Kobordismustheorie*, Diploma thesis 1991, arXiv:math/0405232.