# Higher Algebra I — Fall 2005

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## Problem sheet 5 September 23, 2005

**Problem 1:** Prove *Cauchy's Theorem:* Let $p$ be a prime divisor of the order of a finite group $G$. Then there exists an element $a \in G$ of order $p$.

**Problem 2:** Prove that there is no simple group of order $pqr$, where $p$, $q$ and $r$ are distinct primes.

**Problem 3:** Prove that there is no simple group of order 2907.

**Problem 4:** Let $A$ be a finite abelian group and let $p$ be a prime. The $p^{\text{th}}$-power map is the homomorphism defined by

$$\varphi : A \longrightarrow A, \qquad a \mapsto \varphi(a) = a^p.$$

Let $A^p$ be the image and let $A_p$ be the kernel of $\varphi$, respectively. Prove that $A/A^p \cong A_p$. (Hint: Show that both groups are elementary abelian $p$-groups of the same order. An elementary abelian $p$-group is an abelian group where each element besides the identity has order $p$.)

**Problem 5*:** Prove that there is no simple group of order 1004913. (Hint: Find all possible values for the numbers $n_p$ of $p$-Sylow subgroups and exclude them one by one. Finally, use a permutation representation of degree 819 to exclude the last case.)