Calculus I - Lecture 8 - The derivative function A

Lecture Notes:
http://www.math.ksu.edu/~gerald/math220d/

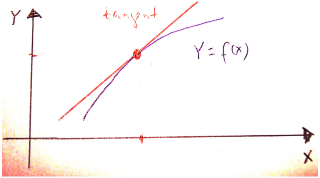
Course Syllabus:
http://www.math.ksu.edu/math220/spring-2014/indexs14.html

Gerald Hoehn (based on notes by T. Cochran)

February 17, 2014

Section 3.2 – Derivative as a function

Last time we saw the geometric and algebraic definition of the derivative of a function $f(x)$ at a point $x = a$



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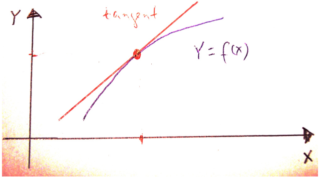

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The **derivative function** $f'(x)$ *of a function* $f(x)$ *is the function which has the value* $f'(a)$ *for* $x = a$*.*

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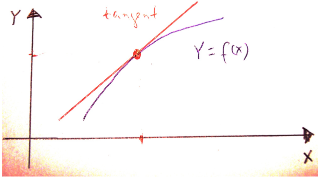

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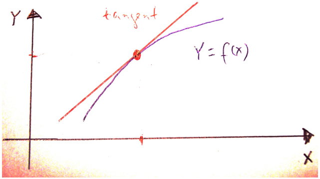

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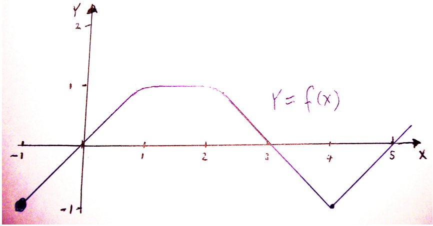
We say $f(x)$ is **differentiable** on (a, b) if it is defined there. If $f'(x)$ exists for all x in the domain of $f(x)$ we simply say $f(x)$ is differentiable.

Example: (Graphical determination of the derivative)

Plot the graph of $f'(x)$ for the function $f(x)$ given by the following graph:



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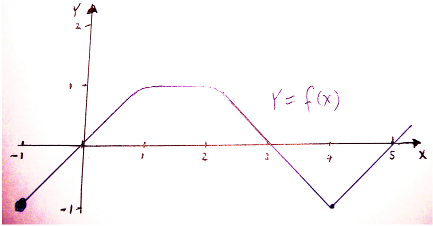

Solution:

x	$-.5$	0	$.5$	1.5	2.5	3	4	5
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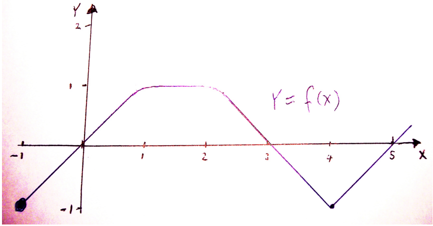

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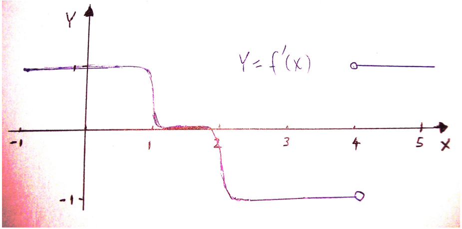

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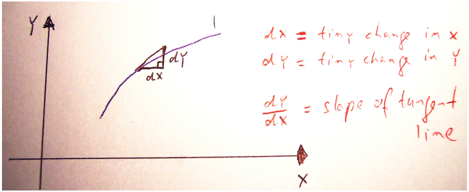

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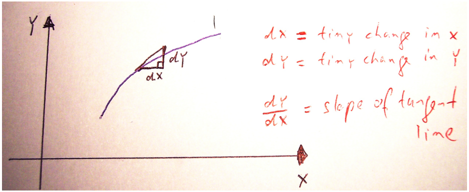

$\frac{\mathrm{d}}{\mathrm{d}x}$ has the operator symbol meaning:
"take the derivative of the function $f(x)$"

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Example: a) Find the derivative of $f(x) = x^2$ using the limit definition, and display the answer in Leibniz notation.

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Rule 1: $\frac{\mathrm{d}}{\mathrm{d}x}\, c = 0$ (c a constant)

Example: a) $\frac{\mathrm{d}}{\mathrm{d}x}\, 17 = 0$ b) $\frac{\mathrm{d}}{\mathrm{d}x}\, \sqrt{2} = 0$

Rule 2: Power Rule. For every real number n

$$\frac{\mathrm{d}}{\mathrm{d}x}\, x^n = n\, x^{n-1} \quad \text{(where } x \text{ is defined)}$$

Example: a) $\frac{\mathrm{d}}{\mathrm{d}x}\, x^7 = 7\, x^{7-1} = 7\, x^6$

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Rule 3: Sum and Difference Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x) \pm g(x)\right) = \frac{\mathrm{d}}{\mathrm{d}x} f(x) \pm \frac{\mathrm{d}}{\mathrm{d}x} g(x)$$

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left(c \cdot f(x)\right) = c \frac{\mathrm{d}}{\mathrm{d}x} f(x) \qquad (c \text{ a constant})$$

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Example:

a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(7\,x^3 - \frac{1}{x} + 5\right)$

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b) $\frac{\mathrm{d}}{\mathrm{d}t}\sqrt{\pi t}$

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Example: Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((x - x^3)(\frac{1}{x} + x^4)\right)$.

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Solution:

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Solution:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x - x^3)(\frac{1}{x} + x^4)\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(1 - x^2 + x^5 - x^7\right)$$

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Rule 5: Exponential Function

$$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x \qquad (\mathrm{e}{=}2.718281\ldots)$$

Example: Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big((x - x^3)(\frac{1}{x} + x^4)\Big)$.

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Rule 5: Exponential Function

$$\frac{\mathrm{d}}{\mathrm{d}x}\,e^x = e^x \qquad (\mathrm{e}{=}2.718281\ldots)$$

The derivative of the exponential function is itself.

Example: Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big((x - x^3)(\frac{1}{x} + x^4)\Big)$.

Solution:

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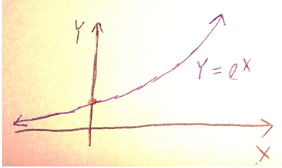
Rule 5: Exponential Function

$$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x \qquad (e=2.718281\ldots)$$

The derivative of the exponential function is itself.

Example: Find the slope of the curve $y = e^x$ at $x = 0$.

Solution:



Example: Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\Big((x - x^3)(\frac{1}{x} + x^4)\Big)$.

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$$\frac{\mathrm{d}}{\mathrm{d}x}\Big((x - x^3)(\frac{1}{x} + x^4)\Big) = \frac{\mathrm{d}}{\mathrm{d}x}(1 - x^2 + x^5 - x^7)$$
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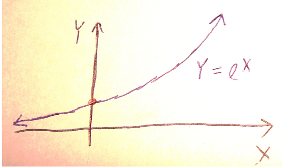


$$\frac{\mathrm{d}}{\mathrm{d}x}\,e^x\Big|_{x=0}$$

Example: Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((x - x^3)(\frac{1}{x} + x^4)\right)$.

Solution:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x - x^3)(\frac{1}{x} + x^4)\right) = \frac{\mathrm{d}}{\mathrm{d}x}(1 - x^2 + x^5 - x^7)$$
$$= -2x + 5\,x^4 - 7x^6$$

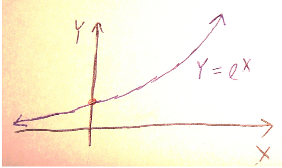
Rule 5: Exponential Function

$$\frac{\mathrm{d}}{\mathrm{d}x}\, e^x = e^x \qquad (e=2.718281...)$$

The derivative of the exponential function is itself.

Example: Find the slope of the curve $y = e^x$ at $x = 0$.

Solution:



$$\frac{\mathrm{d}}{\mathrm{d}x}\, e^x\Big|_{x=0} = e^x\Big|_{x=0}$$

Example: Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left((x-x^3)(\frac{1}{x}+x^4)\right)$.

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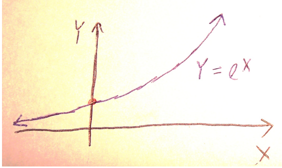

$$\frac{\mathrm{d}}{\mathrm{d}x}e^x\Big|_{x=0} = e^x\Big|_{x=0} = e^0$$

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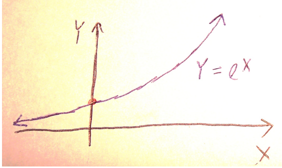

$$\frac{\mathrm{d}}{\mathrm{d}x}\,e^x\Big|_{x=0} = e^x\Big|_{x=0} = e^0 = 1$$

Section 3.3 – Product and Quotient Rule

Rule 6: Product Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\cdot g(x)\right) = \frac{\mathrm{d}}{\mathrm{d}x}f(x)\cdot g(x) + f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}g(x)$$

Section 3.3 – Product and Quotient Rule

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Example:

a) $\frac{\mathrm{d}}{\mathrm{d}x}(x^7 e^x)$

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Example:

a) $\frac{\mathrm{d}}{\mathrm{d}x}(x^7\, e^x) = \frac{\mathrm{d}}{\mathrm{d}x}x^7\cdot e^x + x^7\cdot\frac{\mathrm{d}}{\mathrm{d}x}e^x$

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$= 7x^6 e^x + x^7 e^x$

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b) $\frac{\mathrm{d}}{\mathrm{d}x}((x - x^3)(\frac{1}{x} + x^4))$

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)\cdot g(x)\right) = \frac{\mathrm{d}}{\mathrm{d}x} f(x)\cdot g(x) + f(x)\cdot \frac{\mathrm{d}}{\mathrm{d}x} g(x)$$

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$= (7x^6 + x^7)e^x$

b) $\frac{\mathrm{d}}{\mathrm{d}x}\left((x - x^3)(\frac{1}{x} + x^4)\right)$

$= \frac{\mathrm{d}}{\mathrm{d}x}(x - x^3)\cdot(\frac{1}{x} + x^4) + (x - x^3)\cdot\frac{\mathrm{d}}{\mathrm{d}x}(\frac{1}{x} + x^4)$

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$= (1 - 3x^2)(x^{-1} + x^4) + (x - x^3)(-x^{-2} + 4x^3)$

Section 3.3 – Product and Quotient Rule

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$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = \frac{d}{dx}f(x)\cdot g(x) + f(x)\cdot\frac{d}{dx}g(x)$$

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Example:

a) $\frac{d}{dx}(x^7 e^x) = \frac{d}{dx}x^7\cdot e^x + x^7\cdot\frac{d}{dx}e^x$

$= 7x^6 e^x + x^7 e^x$

$= (7x^6 + x^7)e^x$

b) $\frac{d}{dx}\left((x - x^3)(\frac{1}{x} + x^4)\right)$

$= \frac{d}{dx}(x - x^3)\cdot(\frac{1}{x} + x^4) + (x - x^3)\cdot\frac{d}{dx}(\frac{1}{x} + x^4)$

$= (1 - 3x^2)(x^{-1} + x^4) + (x - x^3)(-x^{-2} + 4x^3)$

$= (x^{-1} - 3x + x^4 - 3x^6) + (-x^{-1} + x + 4x^4 - 4x^6)$

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a) $$\frac{\mathrm{d}}{\mathrm{d}x}(x^7 e^x) = \frac{\mathrm{d}}{\mathrm{d}x} x^7 \cdot e^x + x^7 \cdot \frac{\mathrm{d}}{\mathrm{d}x} e^x$$
$$= 7x^6 e^x + x^7 e^x$$
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$$= (1 - 3x^2)(x^{-1} + x^4) + (x - x^3)(-x^{-2} + 4x^3)$$
$$= (x^{-1} - 3x + x^4 - 3x^6) + (-x^{-1} + x + 4x^4 - 4x^6)$$
$$= -2x + 5x^4 - 7x^6$$

Rule 7: Quotient Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right)=\frac{\frac{\mathrm{d}}{\mathrm{d}x}f(x)\cdot g(x)-f(x)\cdot\frac{\mathrm{d}}{\mathrm{d}x}g(x)}{g(x)^2}$$

Rule 7: Quotient Rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{\mathrm{d}}{\mathrm{d}x} f(x) \cdot g(x) - f(x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} g(x)}{g(x)^2}$$

Example:

a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^2 - 1}{2x + 5}\right)$

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{\mathrm{d}}{\mathrm{d}x} f(x) \cdot g(x) - f(x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} g(x)}{g(x)^2}$$

Example:

a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^2-1}{2x+5}\right)$

$$= \frac{\frac{\mathrm{d}}{\mathrm{d}x}(x^2-1)\cdot(2x+5) - (x^2-1)\cdot\frac{\mathrm{d}}{\mathrm{d}x}(2x+5)}{(2x+5)^2}$$

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$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right) = \frac{\frac{\mathrm{d}}{\mathrm{d}x} f(x) \cdot g(x) - f(x) \cdot \frac{\mathrm{d}}{\mathrm{d}x} g(x)}{g(x)^2}$$

Example:

a) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^2-1}{2x+5}\right)$

$$= \frac{\frac{\mathrm{d}}{\mathrm{d}x}(x^2-1)\cdot(2x+5) - (x^2-1)\cdot\frac{\mathrm{d}}{\mathrm{d}x}(2x+5)}{(2x+5)^2}$$

$$= \frac{2x(2x+5) - (x^2-1)\cdot 2}{(2x+5)^2}$$

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$$= \frac{2x(2x+5) - (x^2-1) \cdot 2}{(2x+5)^2}$$

$$= \frac{2x^2+10x+2}{(2x+5)^2}$$

b) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{t^2 e^t}{1+t^2}\right)$

b) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{t^2e^t}{1+t^2}\right)$

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b) $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{t^2e^t}{1+t^2}\right)$

$$= \frac{\frac{\mathrm{d}}{\mathrm{d}t}(t^2e^t)\cdot(1+t^2) - t^2e^t\cdot\frac{\mathrm{d}}{\mathrm{d}t}(1+t^2)}{(1+t^2)^2} \qquad \text{(quotient first)}$$

$$= \frac{(2te^t + t^2e^t)\cdot(1+t^2) - t^2e^t\cdot 2t}{(1+t^2)^2}$$

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$$= \frac{(2t+t^2+t^4)e^t}{(1+t^2)^2}$$