Calculus I - Lecture 26 - Area and Volume

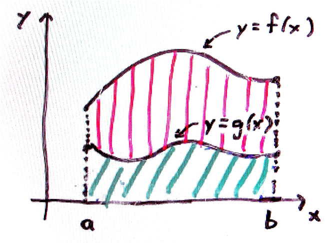
Lecture Notes:
http://www.math.ksu.edu/˜gerald/math220d/

Course Syllabus:
http://www.math.ksu.edu/math220/spring-2014/indexs14.html

Gerald Hoehn (based on notes by T. Cochran)

April 28, 2014

Section 6.1 – Area between curves



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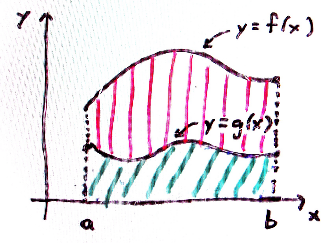

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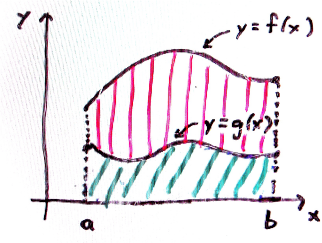

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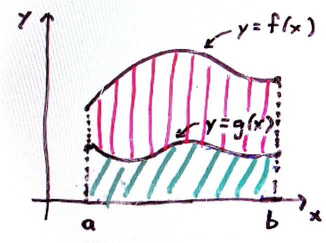

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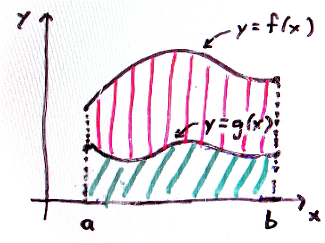

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The calculation is usually simpler if you subtract the functions first.

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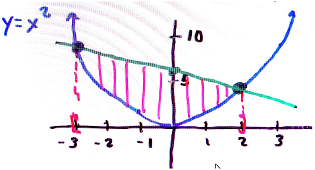

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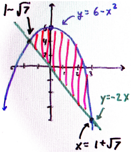

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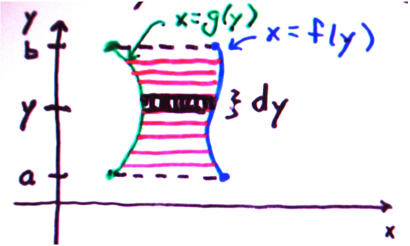

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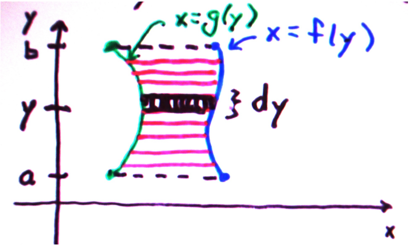

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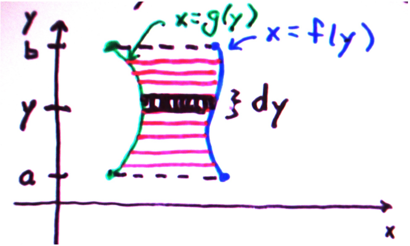

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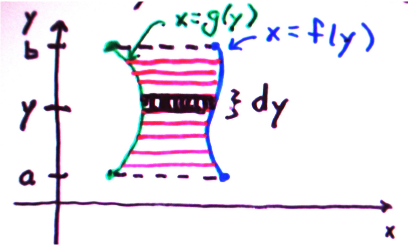

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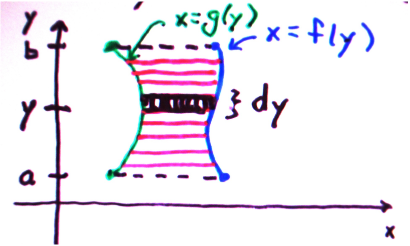

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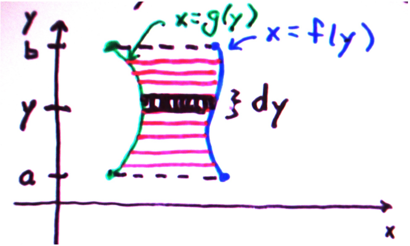

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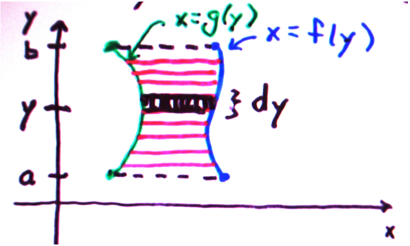

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Same formula with y instead of x.

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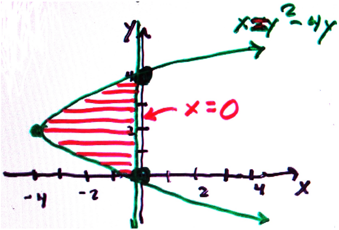

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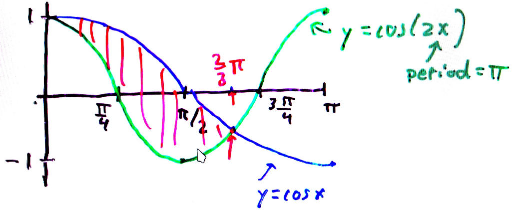

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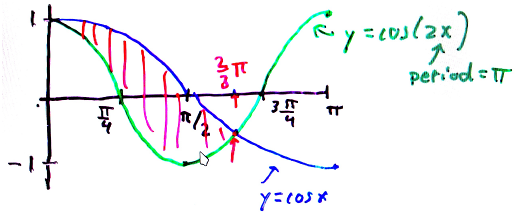

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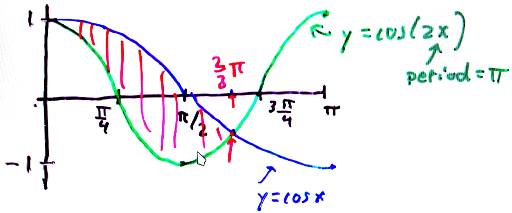

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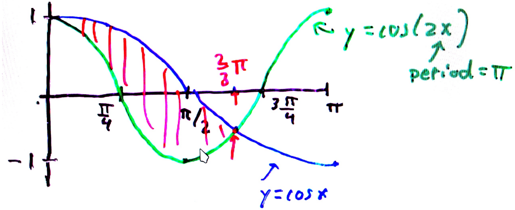

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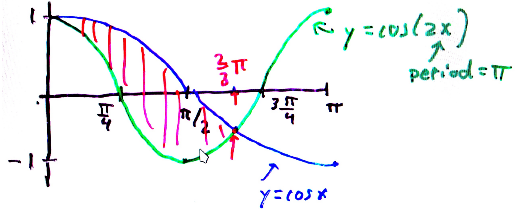

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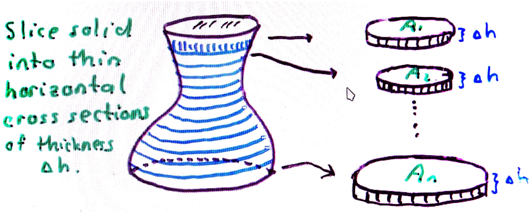
Section 6.2 – Volumes by Cross-Sections

Basic Problem: Determine the volume V of a solid like the one below.



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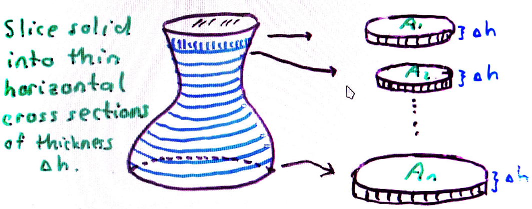

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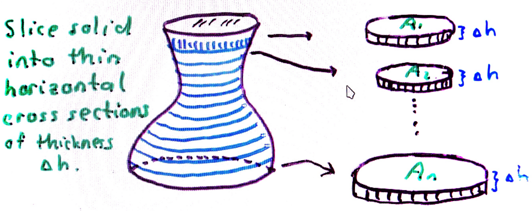

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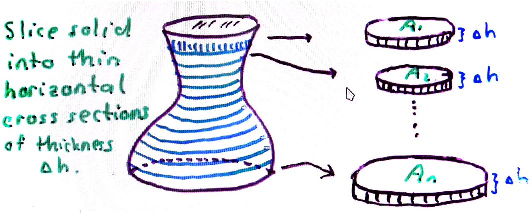

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$$V = \sum_{j=1}^{n} V_j \approx \sum_{j=1}^{n} A_j \, \Delta h$$

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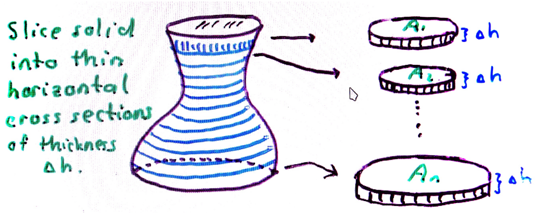

Let A_j = cross sectional area of the $\mathrm{j^{th}}$ slice.

V_j = volume of the $\mathrm{j^{th}}$ slice $\approx A_j \cdot \Delta h$.

$$V = \sum_{j=1}^{n} V_j \approx \sum_{j=1}^{n} A_j \, \Delta h$$

$$V = \lim_{\Delta h \to 0} \sum_{j=1}^{n} A_j \, \Delta h = \int_a^b A \, dh$$

Where $A = A(h)$ is the **cross-sectional area** and h runs from a to b.

Theorem (Volume by Cross-Section Formula)

$$V = \int_a^b A(y)\, dy$$

where $A(y)$ is the area of a cross-section perpendicular to the y-axis.

Example: Find the volume of a cone of height H and radius R.

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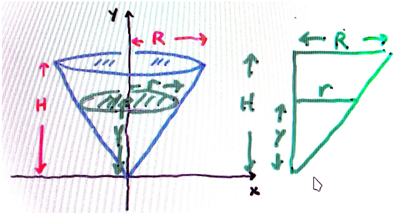

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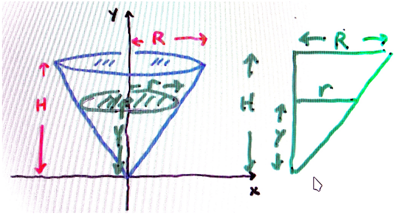

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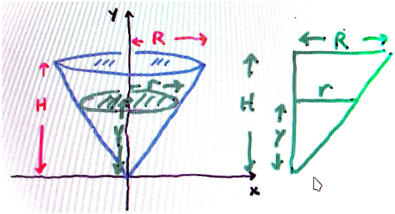

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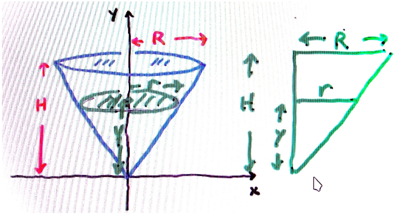

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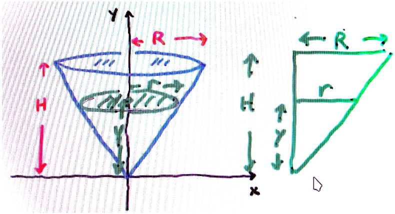

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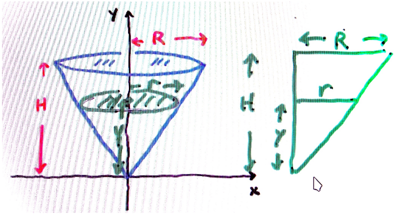

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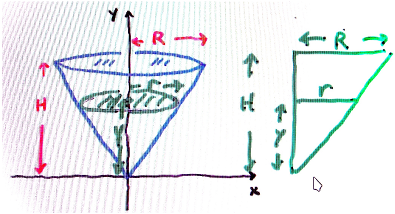

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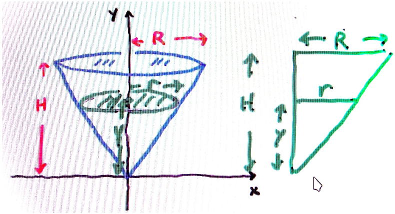

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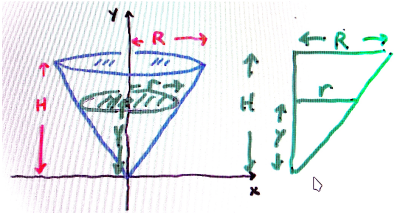

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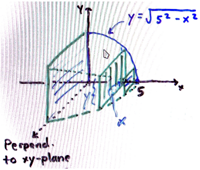

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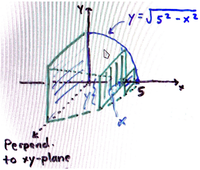

Solution: $y = \sqrt{5^2 - x^2} \;\Leftrightarrow\; x^2 + y^2 = 5^2$, circle of radius 5.

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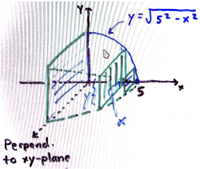

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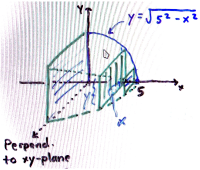

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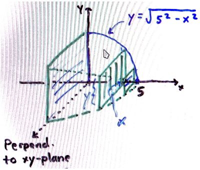

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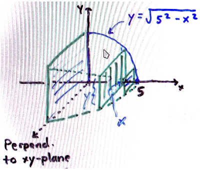

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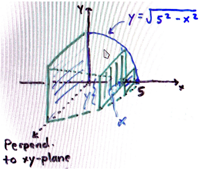

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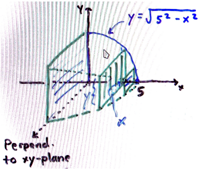

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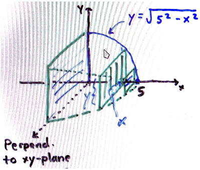

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For some regions, the volume can be calculated in more than one way using cross sections.

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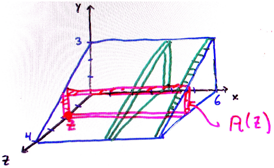

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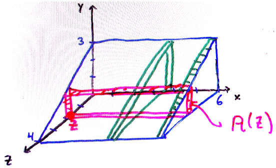

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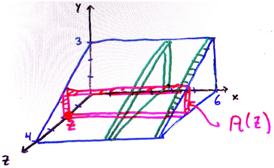

Solution: a) Using cross section perpendicular to x-axis:

Area of green triangle $= \frac{1}{2} \cdot 4 \cdot 3 = 6$.

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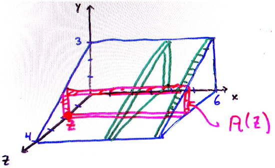

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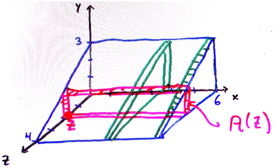

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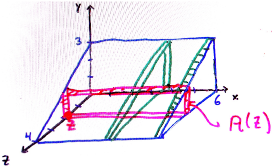

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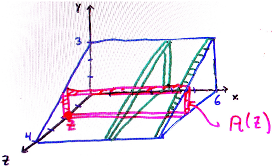

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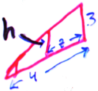
Solution: b) Using cross section perpendicular to z-axis:

Area of red rectangle $= 6 \cdot h = 6 \cdot \frac{3}{4}(4 - z) = \frac{9}{2}(4 - z)$.



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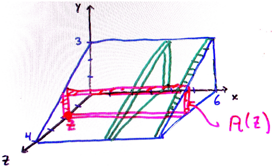

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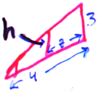

$V = \int_0^4 A(x)\,dx$

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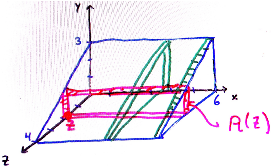

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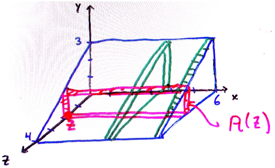

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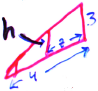

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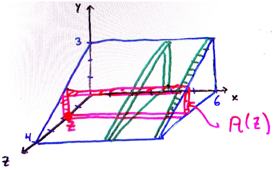

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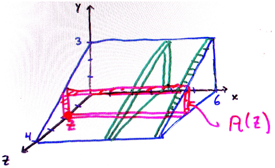

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Solution: b) Using cross section perpendicular to z-axis:

Area of red rectangle $= 6 \cdot h = 6 \cdot \frac{3}{4}(4 - z) = \frac{9}{2}(4 - z)$.

$$V = \int_0^4 A(x)\,dx = \int_0^4 \frac{9}{2}(4-z)\,dx = \frac{9}{2}(4z-\frac{1}{2}z^2)\Big|_0^4 = \frac{9}{2}(16-8) = 36$$