Calculus I - Lecture 25
Net change as Integral of a Rate

Lecture Notes:
http://www.math.ksu.edu/~gerald/math220d/

Course Syllabus:
http://www.math.ksu.edu/math220/spring-2014/indexs14.html

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April 23, 2014

Section 5.7 - Miscellaneous Integrals

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The last integral will be done in Calculus II

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&= \frac{1}{6}\arctan\left(\frac{2}{3}x\right) + C
\end{aligned}$$

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Theorem (Net change formula)

$$\underbrace{\int_a^b r(t)\,dt}_{\text{Integral of the rate of change } r} = \underbrace{s(b) - s(a)}_{\text{Net change in } s \text{ over } [a, b]}$$

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From min. 1 to min. 5 there flows a total of $\frac{124}{3}$ ft^3 into the tank.

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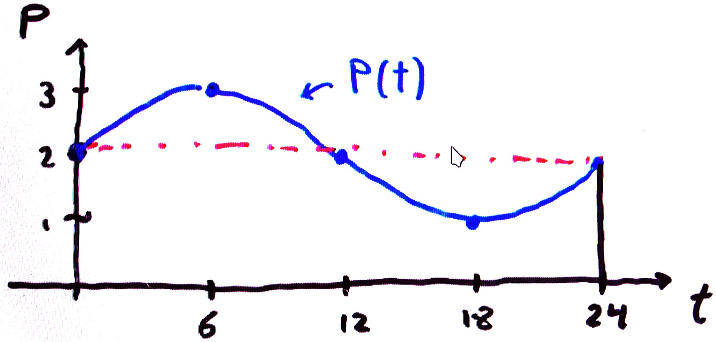
During one day the appliance consumes 48 joules.



P
P(t)
3
2
1
6
12
18
24
t

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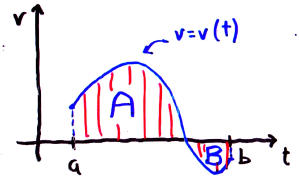

Displacement $= A - B$

Total Distance $= A + B$

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Example: The velocity (in m/sec) of an object is given by $v(t) = t^2 - 2t$. Find the displacement and total distance traveled over the interval $0 \le t \le 5$ (t in sec).

Solution:

a) Displacement $= \int_0^5 (t^2 - 2t)\,dt = \left(\frac{t^3}{3} - t^2\right)\Big|_0^5$

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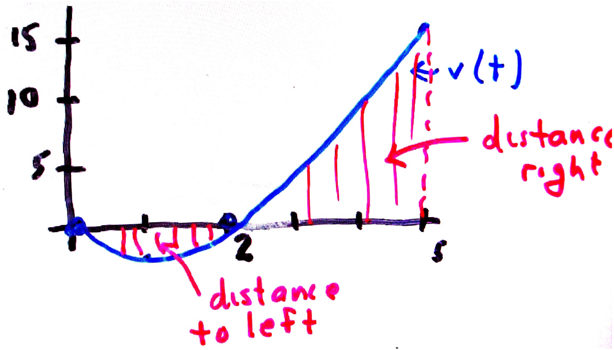
15
10
5
2
5
v(t)
distance right
distance to left

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The velocity changes by $\frac{3}{2}e^2(e^4 - 1)\ \ \mathrm{m/sec}$ over the time interval from one 1 to 3 seconds.

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The total cost of producing 20 items is \$1,666.67.