Calculus I - Lecture 23
Fundamental Theorem of Calculus

Lecture Notes:
http://www.math.ksu.edu/~gerald/math220d/

Course Syllabus:
http://www.math.ksu.edu/math220/spring-2014/indexs14.html

Gerald Hoehn (based on notes by T. Cochran)

April 16, 2014

Section 5.3 - Fundamental Theorem of Calculus I

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Theorem (Fundamental Theorem of Calculus I)

Let $f(x)$ be a continuous function on $[a, b]$. Then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b := F(b) - F(a)$$

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Let $f(x)$ be a continuous function on $[a, b]$. Then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b := F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$.

Note: The result is independent of the chosen antiderivative $F(x)$.

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3. Use of the Riemann sum $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

   (This we will not do in this course.)

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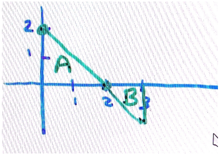
Solution:

1) Areas: 

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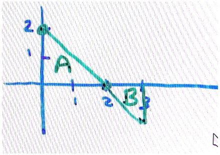
1) Areas: 

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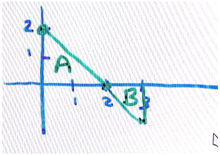
1) Areas: 

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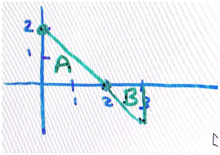
1) Areas: 

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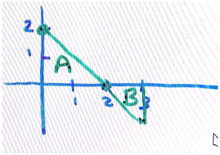
1) Areas: 

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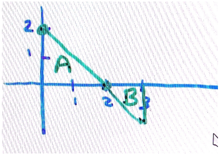
1) Areas: 

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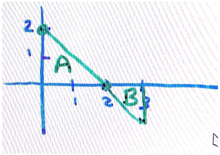
1) Areas: 

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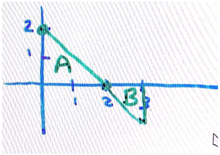

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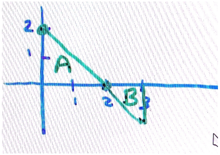
1) Areas: 

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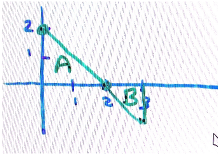

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$$= 6 - \frac{9}{2} = \frac{3}{2}$$

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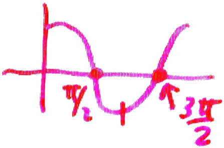

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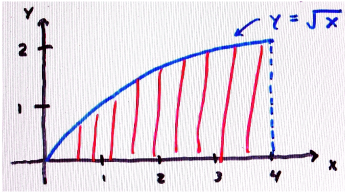

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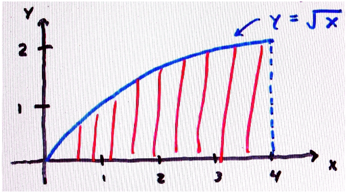

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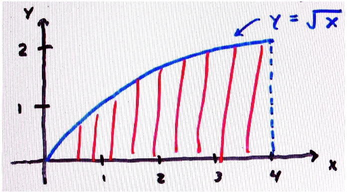

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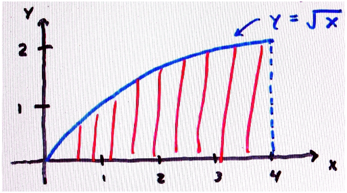

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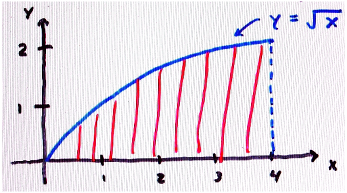

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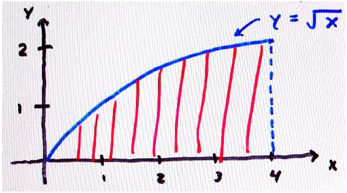

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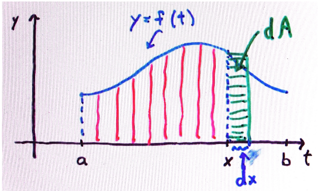
Section 5.4 - Fundamental Theorem of Calculus II

Let $f(t)$ be a continuous function on $[a, b]$.



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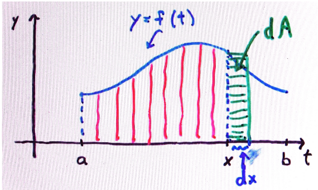

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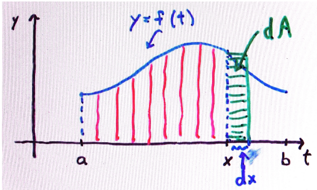

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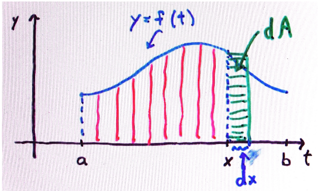

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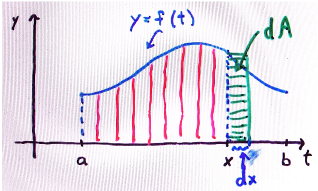

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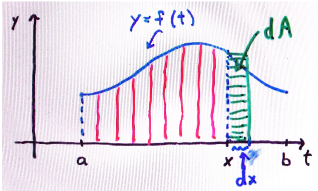

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Let dx = infinitesimal change in x.

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dA = height $\times$ base $= f(x) \cdot dx$.

Thus: $$\frac{dA}{dx} = \frac{d}{dx}\left(\int_a^x f(t)\, dt\right) = f(x).$$

Theorem (Fundamental Theorem of Calculus II)

Let $f(x)$ be a continuous function on $[a, b]$. Then for any x in (a, b) we have

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

where $f(x)$ is the evaluation of $f(t)$ at x.

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Example: Find $\frac{d}{du}\int_{-3}^u \frac{1}{t^2+1}\,dt$

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Example: Find $\displaystyle \frac{d}{dx}\int_2^x e^t \cdot \cos(5t)\,dt$

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Theorem (Fundamental Theorem of Calculus II)

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Flipping the limits of integration

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Let $f(x)$ be a continuous function on $[a, b]$. Then

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