Calculus I - Lecture 16
Minima and Maxima & Mean Value Theorem

Lecture Notes:
http://www.math.ksu.edu/~gerald/math220d/

Course Syllabus:
http://www.math.ksu.edu/math220/spring-2014/indexs14.html

Gerald Hoehn (based on notes by T. Cochran)

March 24, 2014

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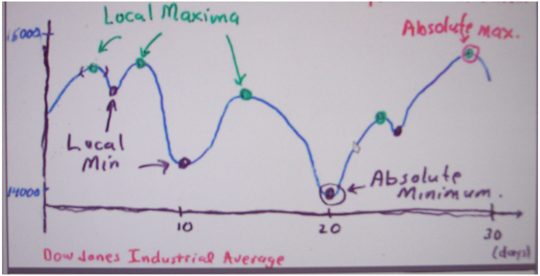

Definition (Absolute Extrema)

Let $f(x)$ be a function defined on on interval I and let $a \in I$.

1. *We say that $f(x)$ has an* **absolute maximum** *at $x = a$ if $f(a)$ is the maximal value of $f(x)$ on I. That is*
$$f(a) \geq f(x) \text{ for all } x \in I.$$
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Definition (Local Extrema)

Let $f(x)$ be a function.

1. *We say that $f(x)$ has an* **local maximum** *at $x = a$ if $f(a)$ is the maximal value of $f(x)$ on some open interval I inside the domain of f containing a.*
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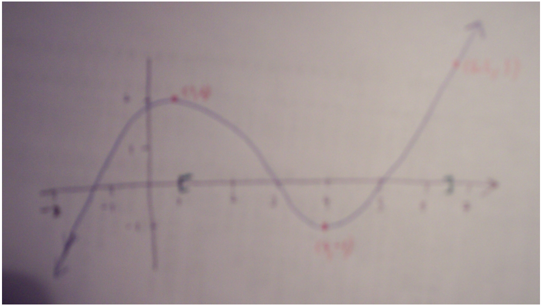
In the above situation the **value** $f(a)$ is called a global (or local) maximum (or minimum).

Example:



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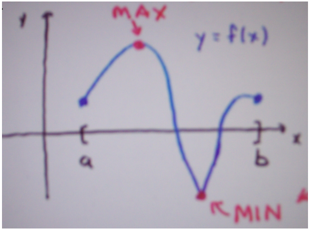

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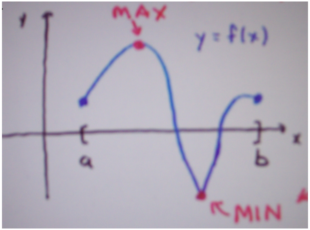

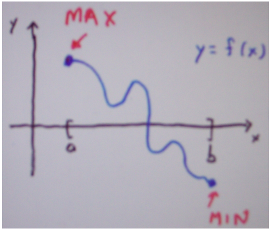

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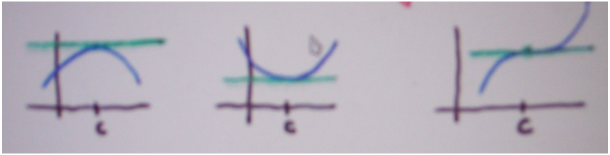

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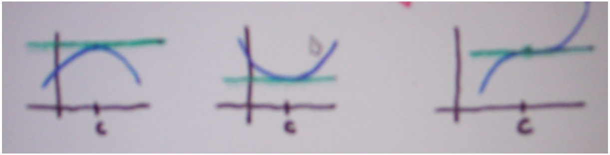
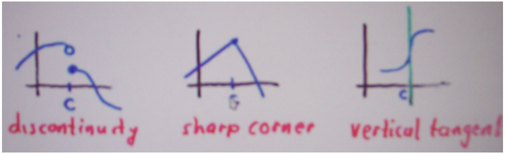


2. *or $f'(c)$ does* **not** *exist.*



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The function $f(x) = x^3$ has no local minima or maxima.

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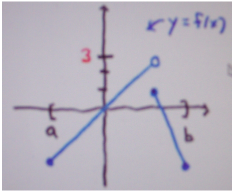

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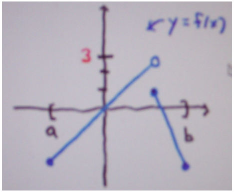

Solution: There is none.

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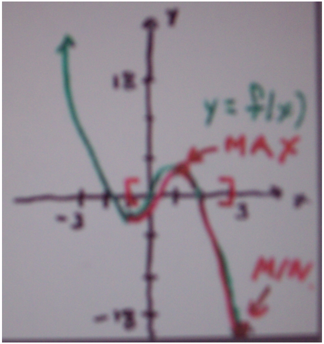
Maximal value is 2 at $x = 1$,
Minimal value is -18 at $x = 3$.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y = f(x)
MAX
-3
3
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Maximal value is $\frac{3}{2\sqrt[3]{2}}$, minimal value is -3.

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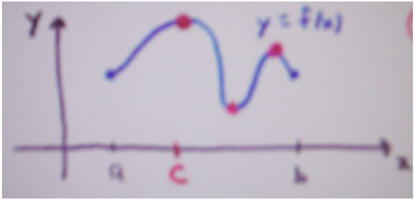

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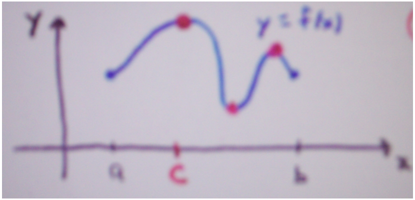

Proof: Follows from the results of the last section!

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$c = \frac{3}{2}$ satisfies the conclusion of Rolle's Theorem.

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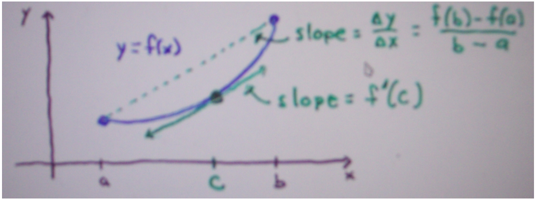

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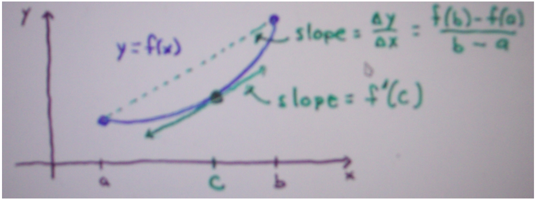

Intuitive version: There is always a time when your instantaneous velocity equals your average velocity.

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