Calculus I - Lecture 1 - Review

Lecture Notes:
http://www.math.ksu.edu/~gerald/math220d/

Course Syllabus:
http://www.math.ksu.edu/math220/spring-2014/indexs14.html

Gerald Hoehn (based on notes by T. Cochran)

January 22, 2014

Overview

CALCULUS

- **Derivatives**
- **Integrals**

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I) **Differential Calculus (Derivatives):** rates of change; speed; slope of a graph; minimum and maximum of functions.

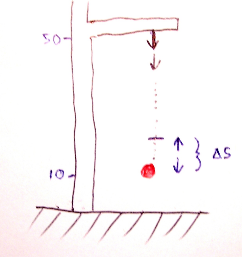
Derivatives measure instantanous changes.

II) **Integral Calculus:** Integrals measure the accumulation of some quantity; the total distance an object has travelled; area under a curve; volume of a region.

An integral can be thought of as a sum of infinitesimal pieces.

Example Derivatives

Example: An apple is observed to drop from a branch 50 ft. above the ground.
Question: At what speed is it travelling when it is 10 ft. above the ground?

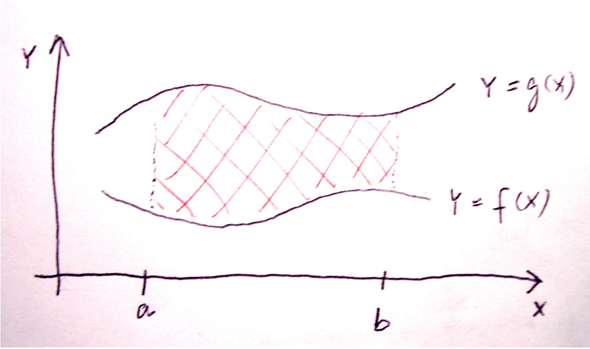


Δs = change in position
Δt = change in in time, to fall Δs
Average speed $= \frac{\Delta s}{\Delta t}$
Speed at instant 10 ft. above $= \lim_{\Delta t \longrightarrow 0} \frac{\Delta s}{\Delta t} = \frac{\mathrm{d}s}{\mathrm{d}t}$

Instantanous Speed.

Example Integrals

Example: Find the area between the curves $y = f(x)$ and $y = g(x)$ over the interval $[a, b]$.



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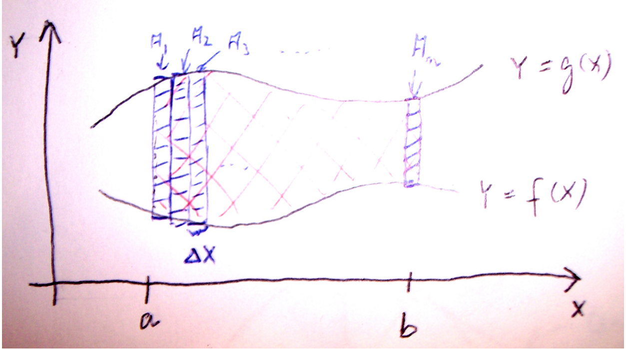

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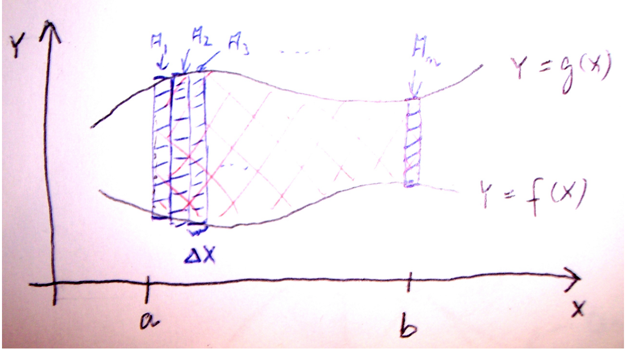


Total Area $\approx$ sum of areas of slices $= A_1 + A_2 + A_3 + \cdots + A_n$.

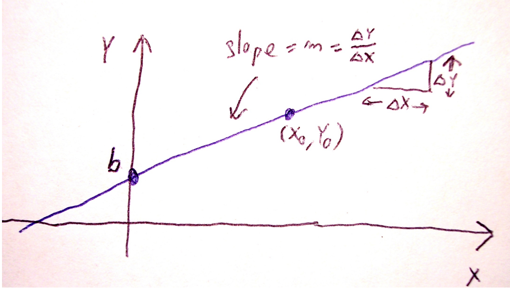
$$\text{Total Area} = \lim_{\Delta x \longrightarrow 0} (A_1 + \cdots + A_n) = \int_a^b (g(x) - f(x))\,\mathrm{d}x$$

Definite Integral.

Success in Calc I begins with a solid foundation in **Algebra and Trigonometry**!

We will review some of the most important concepts.

Equations of Lines



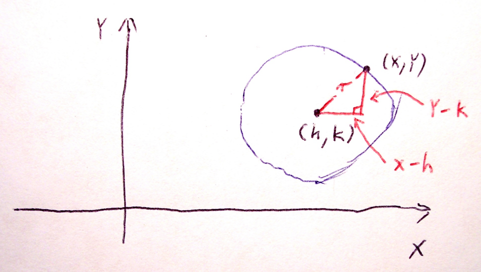
First degree equation in x, y.

1. **Point-Slope Form:** $y - y_0 = m(x - x_0) \Leftrightarrow \frac{y-y_0}{x-x_0} = m$
2. **Slope-Intercept Form:** $y = mx + b$ (b is y-intercept)
3. **Standard Form:** $Ax + By = C$

Facts:

- Parallel lines have the same slope.
- The slope of a perpendicular line is $-\frac{1}{m}$ (negative-reciprocal).

Equations of Circles



2nd degree equation in x, y.

$(x-h)^2 + (y-k)^2 = r^2$ (by Pythagorean Theorem)

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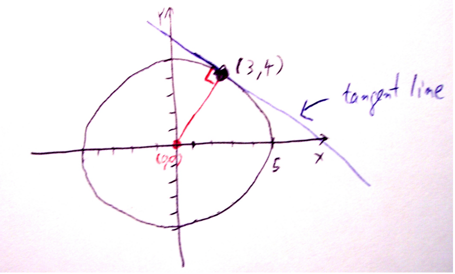

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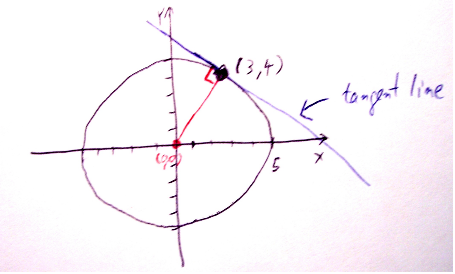

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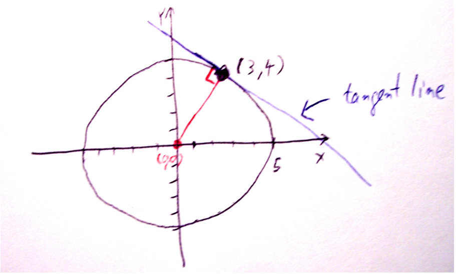

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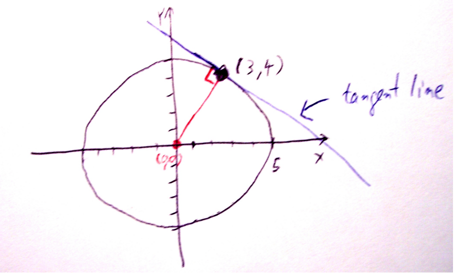

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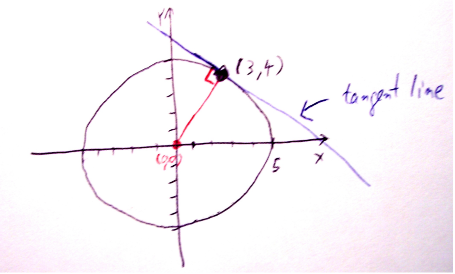

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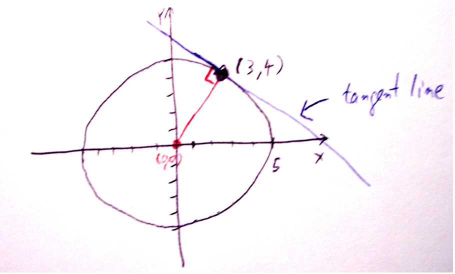

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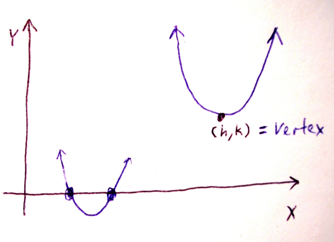
$$\Rightarrow y = -\frac{3}{4}x + \frac{9}{4} + 4 = -\frac{3}{4}x + \frac{25}{4}.$$

Equations of Parabolas



2nd degree equation in x <u>or</u> y.

1. **Vertex-Point Form:** $y = a(x-h)^2 + k$, $a > 0$: $\cup$, $a < 0$: $\cap$.
2. **Standard Form:** $y = ax^2 + bx + c$

x-**intercepts:** These are the solutions of the quadratic equation

$$ax^2 + bx + c = 0. \qquad (*)$$

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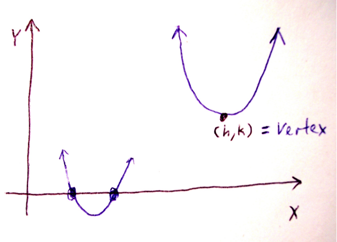

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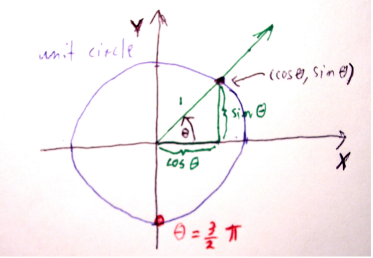
Quadratic Formula: The solutions of (*) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (a \neq 0).$$

Fact: If $b^2 - 4ac < 0$ then (*) has no real solutions, that is, the parabola has no x-intercepts.

Trigonometric Functions

2π radians $= 360°$ $\theta =$ theta



$\sin(\theta) = y$-coord. of point on unit circle
$\cos(\theta) = x$-coord. of point on unit circle
$\tan(\theta) = \frac{\sin\theta}{\cos\theta}$, $\cot(\theta) = \frac{\cos\theta}{\sin\theta}$
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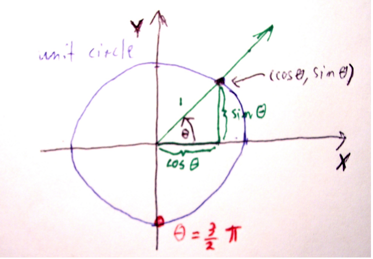

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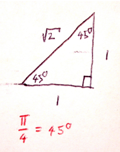
Example:

$\sin(\frac{3}{2}\pi) = -1$, $\qquad \cos(\frac{3}{2}\pi) = 0$

$\tan(\frac{3}{2}\pi) = \frac{-1}{0}$ (undefined!), $\qquad \cot(\frac{3}{2}\pi) = \frac{0}{-1} = 0$

Standard angles

I) 

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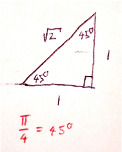
I) 

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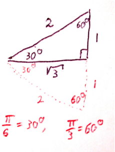
II) 

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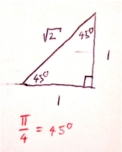
I) 

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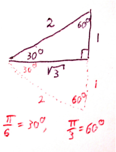
II) 

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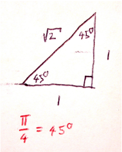
I) 

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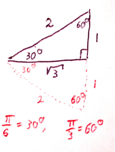
II) 

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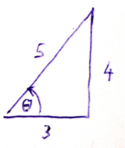

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Exponential and Natural Logarithm Functions

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$$e^x = \text{exponential function (with base } e\text{)}$$

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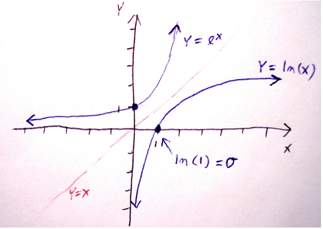

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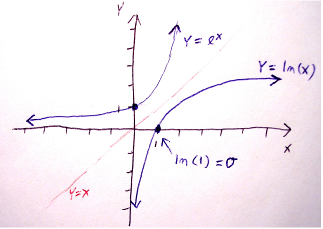

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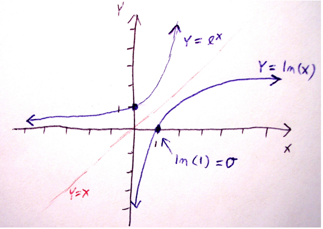

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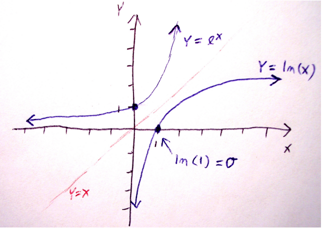

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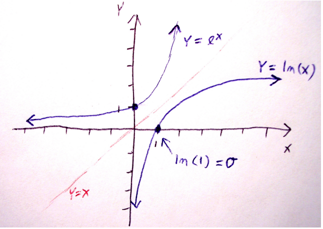

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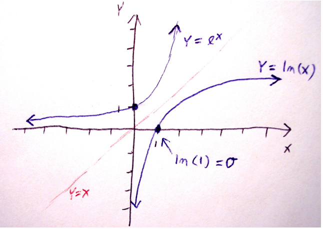

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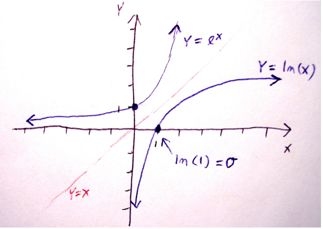

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- $e^{\ln x} = x$, for $x > 0$ $\quad \Rightarrow$ e^x and $\ln x$ are **inverse functions**