

Calculus I - Lecture 27

Volume of Bodies of Revolution

Lecture Notes:

<http://www.math.ksu.edu/~gerald/math220d/>

Course Syllabus:

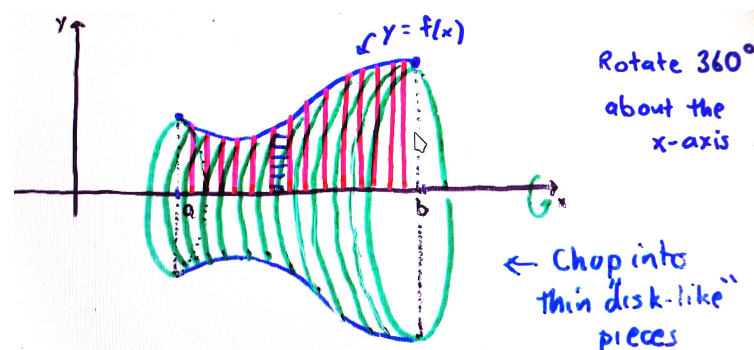
<http://www.math.ksu.edu/math220/spring-2014/indexs14.html>

Gerald Hoehn (based on notes by T. Cochran)

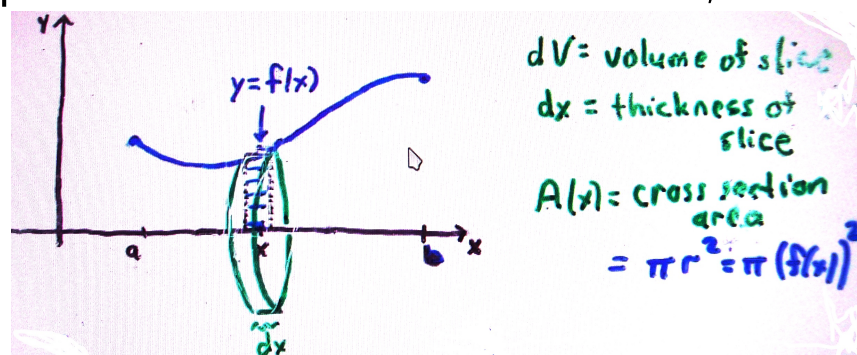
February 5, 2014

Section 6.3 – Volumes of Revolution

Goal: Find the volume of a solid obtained by rotating a region in the xy -plane about an axis.



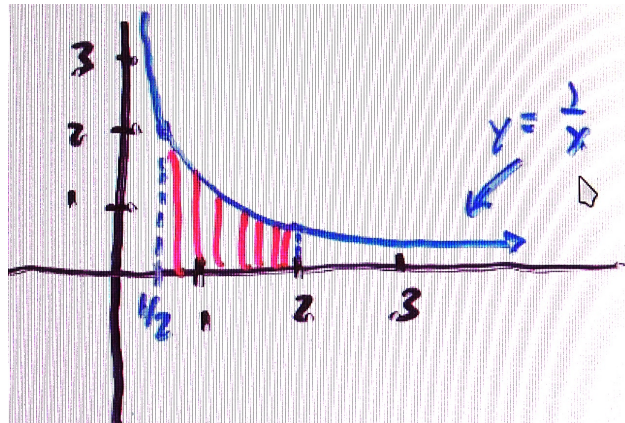
Consider a typical slice located at the value x ;



$$V = \int dV = \int_a^b A(x) dx = \int_a^b \pi f^2(x) dx.$$

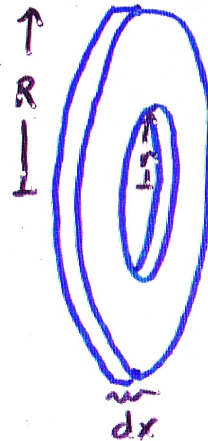
Example: Find the volume of the solid obtained by rotating the region bounded by $y = \frac{1}{x}$, $y = 0$, $x = \frac{1}{2}$ and $x = 2$ about the x -axis.

Solution:

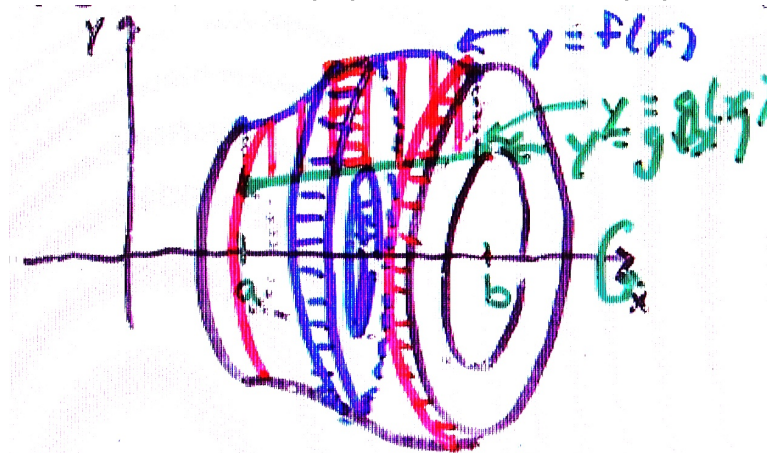


$$\begin{aligned} V &= \int_{1/2}^2 \pi (f(x))^2 dx = \int_{1/2}^2 \pi \left(\frac{1}{x} \right)^2 dx \\ &= \pi \int_{1/2}^2 x^{-2} dx = \pi \cdot \frac{x^{-1}}{-1} \Big|_{1/2}^2 \\ &= -\pi \cdot \frac{1}{2} - \left(-\pi \cdot \frac{1}{1/2} \right) = -\frac{1}{2}\pi + 2\pi = \frac{3}{2}\pi. \end{aligned}$$

Method of Washers: A washer is a disk with a hole punched in it.



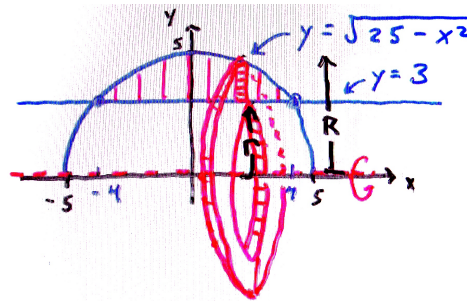
Rotate region between $y = f(x)$ and $y = g(x)$ about the x -axis.



$$V = \int_a^b A \, dx = \int_a^b \pi (R^2 - r^2) \, dx = \int_a^b \pi (f(x)^2 - g(x)^2) \, dx.$$

Example: Rotate the region between $y = \sqrt{25 - x^2}$ and $y = 3$ about the x -axis and find the volume of the resulting body.

Solution:



Intersection points:

$$\sqrt{25 - x^2} = 3 \Leftrightarrow 25 - x^2 = 9 \Leftrightarrow x^2 = 16 \Leftrightarrow x = \pm 4$$

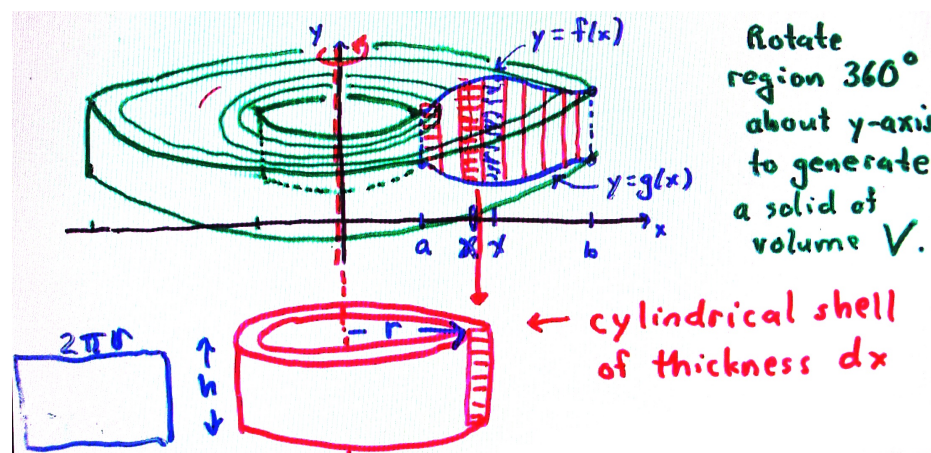
$$V = \int_{-4}^4 \pi(R^2 - r^2) dx = \int_{-4}^4 \pi((\sqrt{25 - x^2})^2 - 3^2) dx$$

$$= \int_{-4}^4 \pi(25 - x^2 - 9) dx = \int_{-4}^4 \pi(16 - x^2) dx$$

$$= \pi \left(16x - \frac{x^3}{3} \right) \Big|_{-4}^4 = \pi \left(64 - \frac{64}{3} \right) - \pi \left(-64 - \frac{-64}{3} \right)$$

$$= \pi \cdot 2 \cdot \frac{2}{3} \cdot 64 = \frac{256}{3} \pi$$

Section 6.4 – Volumes by Cylindrical Shells



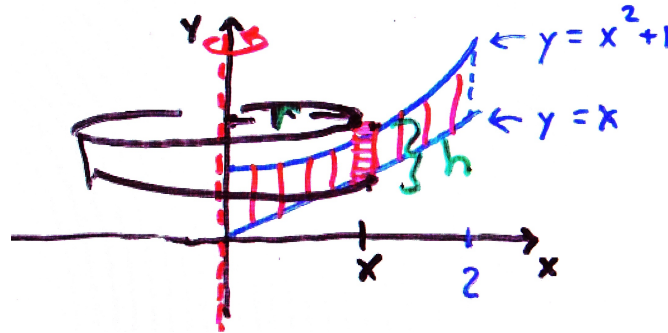
$$dV = \text{volume of shell} = \underbrace{2\pi r h}_{\text{area of shell}} \cdot \underbrace{dx}_{\text{thickness}}$$

$$V = \int dV = \int_a^b 2\pi r h \, dx$$

Since $r = x$ and $h = f(x) - g(x)$ we get

$$V = \int_a^b 2\pi x (f(x) - g(x)) \, dx$$

Example: Rotate the region below about the y-axis and determine the volume of the resulting body.



Solution:

$$dV = 2\pi rh \, dx = 2\pi x((x^2 + 1) - x) \, dx$$

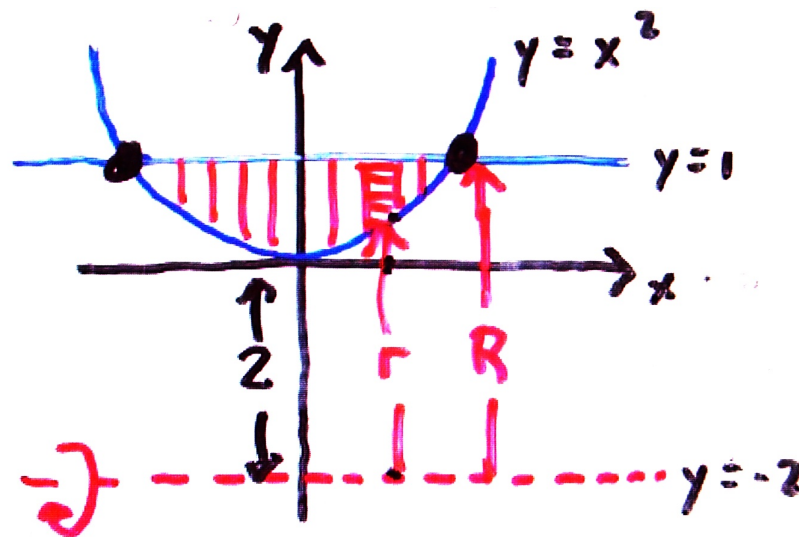
$$V = \int_0^2 2\pi x(x^2 + 1 - x) \, dx$$

$$= 2\pi \int_0^2 (x^3 - x^2 + x) \, dx$$

$$= 2\pi \left(\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} \right) \Big|_0^2 = 2\pi \left(4 - \frac{8}{3} + 2 \right) = \frac{20}{3}\pi$$

Example: Rotate the region bounded by $y = x^2$ and $y = 1$ about the axis $y = -2$ and set up the integral for the volume of the resulting body by a) using washers and b) using shells.

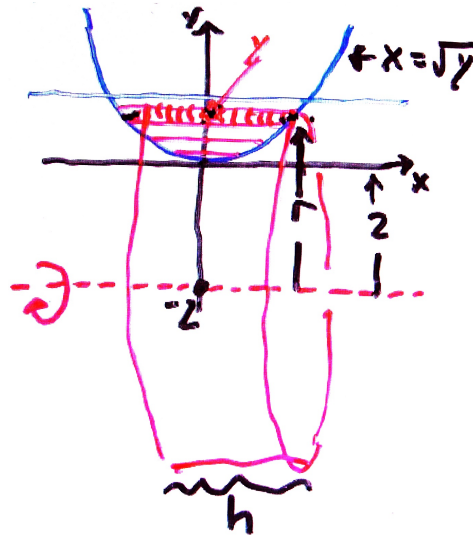
Solution: a)



$$dV = \text{washer volume} = \pi(R^2 - r^2) dx = \pi(3^2 - (2 + x^2)^2) dx$$

$$V = \int dV = \int_{-1}^1 \pi(9 - (2 + x^2)^2) dx$$

Solution: b)



$$dV = \text{volume cyl. shell} = 2\pi r h \, dy = 2\pi(2 + y)(2\sqrt{y}) \, dy$$

$$V = \int dV = \int_0^1 2\pi(2 + y)(2\sqrt{y}) \, dy$$

Teaching Evaluation

Please use a pencil

Please put on top of chart:

Name of instructor:	Gerald Hoehn
Course:	MATH 220
Course number:	12326
Time:	9:30 a.m.

Next Week: Review by Julie Lang