Calculus I - Lecture 1 - Review

Lecture Notes:
http://www.math.ksu.edu/~gerald/math220d/

Course Syllabus:
http://www.math.ksu.edu/math220/spring-2014/indexs14.html

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Overview

CALCULUS

- **Derivatives**
- **Integrals**

I) **Differential Calculus (Derivatives):** rates of change; speed; slope of a graph; minimum and maximum of functions.

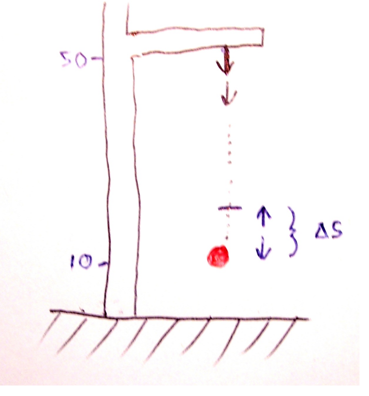
Derivatives measure instantanous changes.

II) **Integral Calculus:** Integrals measure the accumulation of some quantity; the total distance an object has travelled; area under a curve; volume of a region.

An integral can be thought of as a sum of infinitesimal pieces.

Example Derivatives

Example: An apple is observed to drop from a branch 50 ft. above the ground.
Question: At what speed is it travelling when it is 10 ft. above the ground?

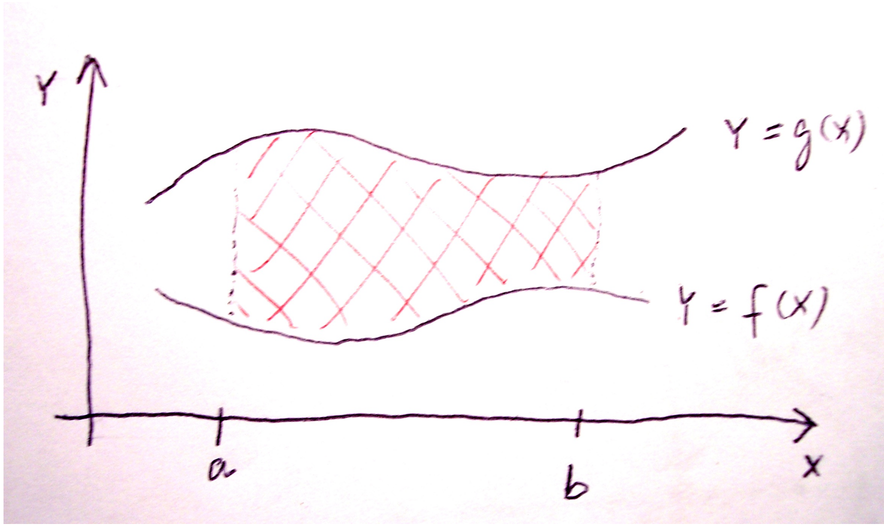


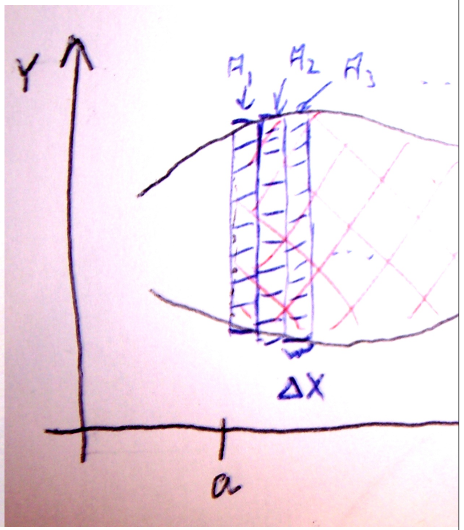
Δs = change in position
Δt = change in in time, to fall Δs
Average speed $= \frac{\Delta s}{\Delta t}$
Speed at instant 10 ft. above $= \lim\limits_{\Delta t \longrightarrow 0} \frac{\Delta s}{\Delta t} = \frac{\mathrm{d}s}{\mathrm{d}t}$

Instantanous Speed.

Example Integrals

Example: Find the area between the curves $y = f(x)$ and $y = g(x)$ over the interval $[a, b]$.





Total Area $\approx$ sum of areas of slices $= A_1 + A_2 + A_3 + \cdots + A_n$.

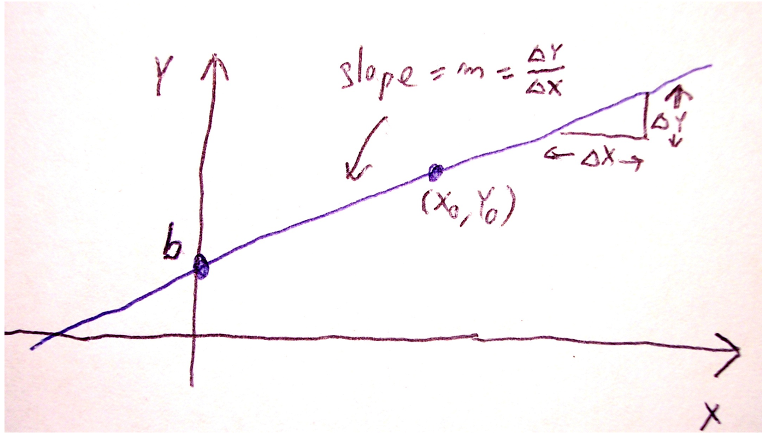
Total Area $= \lim_{\Delta x \longrightarrow 0} (A_1 + \cdots + A_n) = \int_a^b (g(x) - f(x))\,\mathrm{d}x$

Definite Integral.

Success in Calc I begins with a solid foundation in
Algebra and Trigonometry!

We will review some of the most important concepts.

Equations of Lines



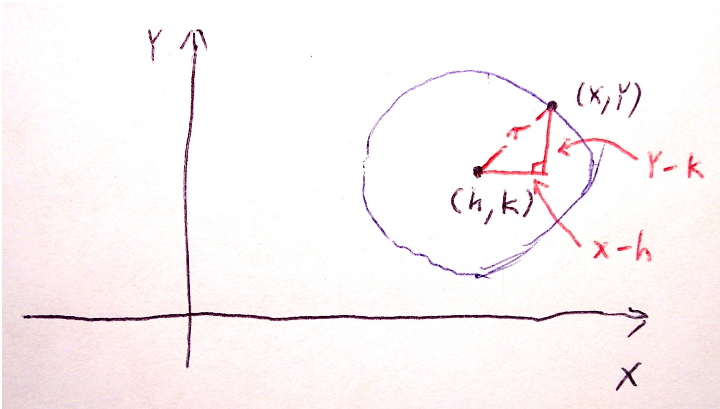
First degree equation in x, y.

1. **Point-Slope Form:** $y - y_0 = m(x - x_0) \Leftrightarrow \frac{y-y_0}{x-x_0} = m$
2. **Slope-Intercept Form:** $y = mx + b$ (b is y-intercept)
3. **Standard Form:** $Ax + By = C$

Facts:

- Parallel lines have the same slope.
- The slope of a perpendicular line is $-\frac{1}{m}$ (negative-reciprocal).

Equations of Circles

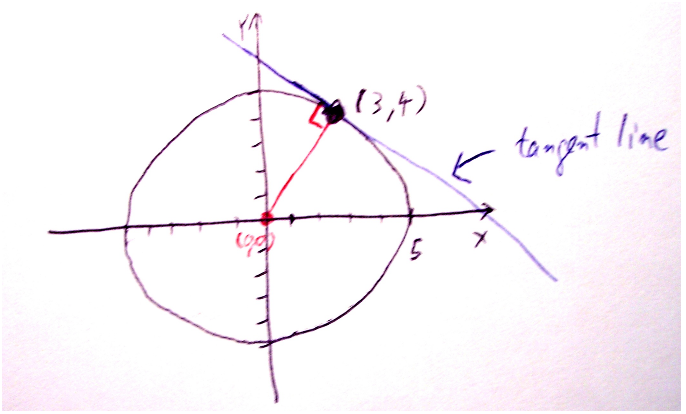


2nd degree equation in x, y.

$(x-h)^2 + (y-k)^2 = r^2$ (by Pythagorean Theorem)

Example: Find the equation of the line tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$ in slope-intercept form.

Solution:



slope red line $= \frac{\Delta y}{\Delta x} = \frac{4-0}{3-0} = \frac{4}{3}$

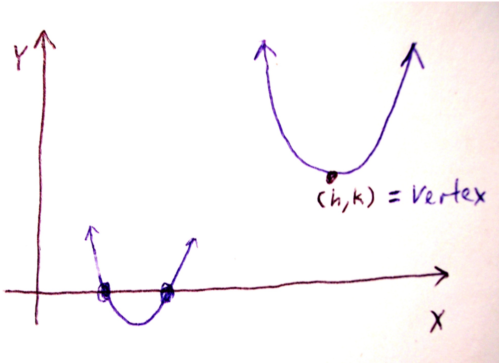
$m =$ slope tangent line $= -\frac{3}{4}$

pt.-slope: $y - y_0 = m(x - x_0)$

$$y - 4 = -\frac{3}{4}(x - 3)$$

$$\Rightarrow y = -\frac{3}{4}x + \frac{9}{4} + 4 = -\frac{3}{4}x + \frac{25}{4}.$$

Equations of Parabolas



2nd degree equation in x or y.

1. **Vertex-Point Form:** $y = a(x-h)^2 + k$, $a > 0$: $\cup$, $a < 0$: $\cap$.
2. **Standard Form:** $y = ax^2 + bx + c$

x**-intercepts:** These are the solutions of the quadratic equation

$$ax^2 + bx + c = 0. \qquad (*)$$

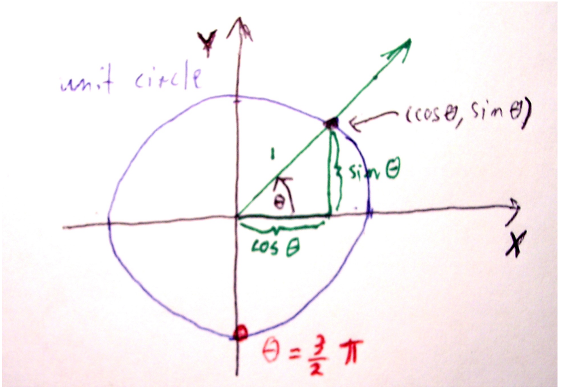
Quadratic Formula: The solutions of (*) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (a \neq 0).$$

Fact: If $b^2 - 4ac < 0$ then (*) has no real solutions, that is, the parabola has no x-intercepts.

Trigonometric Functions

2π radians $= 360°$ $\theta =$ theta



$\sin(\theta) = y$-coord. of point on unit circle
$\cos(\theta) = x$-coord. of point on unit circle
$\tan(\theta) = \frac{\sin\theta}{\cos\theta}$, $\cot(\theta) = \frac{\cos\theta}{\sin\theta}$
$\sec(\theta) = \frac{1}{\cos\theta}$, $\csc(\theta) = \frac{1}{\sin\theta}$

Pythagorean relation: $\sin^2\theta + \cos^2\theta = 1$

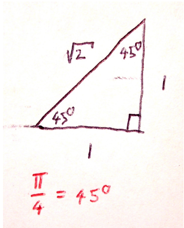
Example:

$\sin(\frac{3}{2}\pi) = -1$, $\cos(\frac{3}{2}\pi) = 0$

$\tan(\frac{3}{2}\pi) = \frac{-1}{0}$ (undefined!), $\cot(\frac{3}{2}\pi) = \frac{0}{-1} = 0$

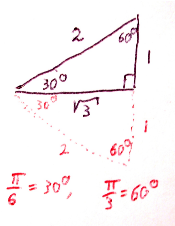
Standard angles

I)



$$\begin{aligned} \sin(\tfrac{\pi}{4}) &= \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \cos(\tfrac{\pi}{4}) &= \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{aligned}$$

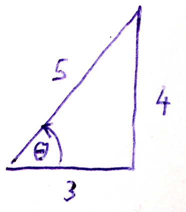
II)



$$\begin{aligned} \sin(\tfrac{\pi}{6}) &= \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} \\ \cos(\tfrac{\pi}{6}) &= \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \\ \sin(\tfrac{\pi}{3}) &= \frac{\sqrt{3}}{2} \\ \cos(\tfrac{\pi}{3}) &= \frac{1}{2} \end{aligned}$$

Example: If $\tan\theta = \frac{4}{3}$ and $\sin\theta > 0$, find $\cos\theta$.

Solution:



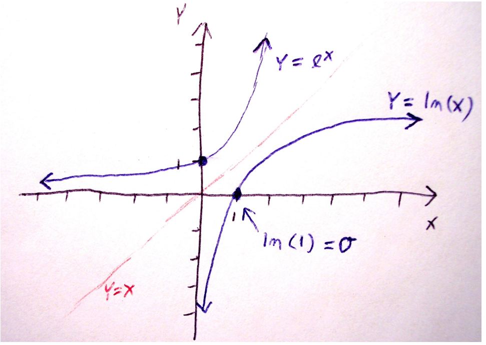
$$\begin{aligned} \tan\theta &= \frac{\text{opp.}}{\text{adj.}} = \frac{4}{3} \\ \cos\theta &= \frac{\text{adj.}}{\text{hyp.}} = \frac{3}{5} \\ \sin\theta &= \frac{\text{opp.}}{\text{hyp.}} = \frac{4}{5}. \end{aligned}$$

Exponential and Natural Logarithm Functions

$$e = 2.718281828\ldots = \text{natural log base}$$

$$e^x = \text{exponential function (with base } e\text{)}$$

$$\ln(x) = \log_e(x) = \text{natural logarithm}$$



Facts:

- $\ln(x)$ is only defined for $x > 0$
- $\ln(xy) = \ln x + \ln y, \quad x,\, y > 0$
- $\ln(x^s) = s \ln x$
- $\ln(e^x) = x$, for all x $\quad \Rightarrow$
- $e^{\ln x} = x$, for $x > 0$ $\quad \Rightarrow$ e^x and $\ln x$ are **inverse functions**